

Hadronic Three Jet Production at Next-to-Leading Order¹

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Abstract

I present results of a next-to-leading order calculation of three jet production at hadron colliders. This calculation will have many applications. In addition to computing three-jet observables (spectra, mass distributions), this calculation permits the first next-to-leading order studies (at hadron colliders) of jet and event shape variables.

1 Introduction

One of the difficulties in interpreting experimental results is in assessing the uncertainty to be associated with the theoretical calculation. In QED and the weak interactions, one generally has confidence in the accuracy of leading order (LO) calculations because the couplings are sufficiently weak that higher order corrections are small. In QCD, however, the coupling is quite strong and it is difficult to obtain a reliable estimate of the theoretical uncertainty.

One typically characterizes theoretical uncertainty by the dependence on the renormalization scale μ . Since one doesn't actually know how to choose

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μ or even a range of μ , the uncertainty associated with scale dependence is somewhat arbitrary. One motivation for performing next-to-leading order (NLO) calculations is to reduce the scale dependence associated with the calculation.

However, this is not the only benefit of an NLO calculation. There are times when the LO calculation is a bad estimator of the physical process. It may be that leading order kinematics artificially forbids the most important physical process. It could also be that the NLO corrections are simply large. Even if the overall NLO correction is relatively small, there may be regions of phase space, where NLO corrections are large. It is only in those regions of phase space where the NLO corrections are well behaved (as determined by the ratio of the NLO to LO terms) that one has confidence in the reliability of the calculation and can begin to believe the uncertainty estimated from scale dependence and it is only when one has a reliable estimate of the theoretical uncertainty that comparisons to experiment are meaningful.

2 Methods

The NLO three jet calculation consists of two parts: two to three parton processes at one-loop (the virtual terms) and two to four parton processes (the real emission terms) at tree-level. Both of these contributions are infrared singular; only the sum of the two is infrared finite and meaningful. The virtual contributions are infrared singular because of loop momenta going on-shell. The real emission contributions are singular when two partons become collinear or when a gluon becomes very soft. The Kinoshita-Lee-Nauenberg theorem [1] guarantees that the infrared singularities cancel for sufficiently inclusive processes when the real and virtual contributions are combined.

The parton sub-processes involved are $gg \rightarrow ggg$ [2], $\bar{q}q \rightarrow ggg$ [3], $\bar{q}q \rightarrow \bar{Q}Qg$ [4], and processes related to these by crossing symmetry, all computed to one-loop, and $gg \rightarrow gggg$, $\bar{q}q \rightarrow gggg$, $\bar{q}q \rightarrow \bar{Q}Qgg$, and $\bar{q}q \rightarrow \bar{Q}Q\bar{Q}'Q'$ and the crossed processes computed at tree-level.

In order to implement the kinematic cuts necessary to compare a calculation to experimental data one must compute the cross section numerically. Thus, it is not sufficient to know that the singularities drop out in the end, we must find a way of canceling them before we start the calculation. Several different methods of implementing this infrared cancellation have been successfully employed in various NLO calculations. The method we use is

the “subtraction improved” phase space slicing method [7]. Phase space slicing [5, 6] uses a resolution criterion s_{\min} , which is a cut on the two parton invariant masses,

$$s_{ij} = 2E_i E_j (1 - \cos \theta_{ij}). \quad (1)$$

If partons i and j have $s_{ij} > s_{\min}$ they are said to be resolved from one another. (Which is not to say that a jet clustering algorithm will not put them into the same jet.) If $s_{ij} < s_{\min}$ partons i and j are said to be unresolvable. One advantage of the s_{\min} criterion is that it simultaneously regulates both soft ($E_i \rightarrow 0$ or $E_j \rightarrow 0$) and collinear ($\cos \theta_{ij} \rightarrow 1$) emission. In the rearrangement of terms, the infrared region of phase space is where any two parton invariant mass is less than s_{\min} . These regions are sliced out of the full two-to-four body phase space, partially integrated and then added to the two-to-three body integral.

Because the infrared integral is bounded by s_{\min} , both the two-to-three and two-to-four body integrations are logarithmically dependent on s_{\min} . Since s_{\min} is an arbitrary parameter the sum of the two contributions must be s_{\min} independent. Thus, we have rearranged the calculation, trading a cancellation of infrared poles for a cancellation of logarithms of s_{\min} . The demonstration of s_{\min} independence implies that we have correctly implemented the infrared cancellation.

3 Results

The results shown below were computed for the following kinematic conditions: the $\bar{p}p$ center of mass energy was 1800 GeV; at least one jet was required to have more than 100 GeV of transverse energy (E_T), while two more jets were required to have more than 50 GeV of transverse energy. All three jets were required to lie in the pseudorapidity range $-4.0 < \eta_J < 4.0$.

The first test of the calculation is to demonstrate s_{\min} independence of the cross section. In figure 1, the next-to-leading order cross section is computed for sixteen values of s_{\min} between 1 and 40 GeV². We see that the computed cross section is stable over a wide range of s_{\min} . The next-to-leading order calculations were all performed using CTEQ3M parton distributions. The renormalization and factorization scales were chosen to be $\mu = 100$ GeV.

For comparison, the leading order calculation (computed using the CTEQ3L parton distributions but all other parameters the same) is shown as a solid

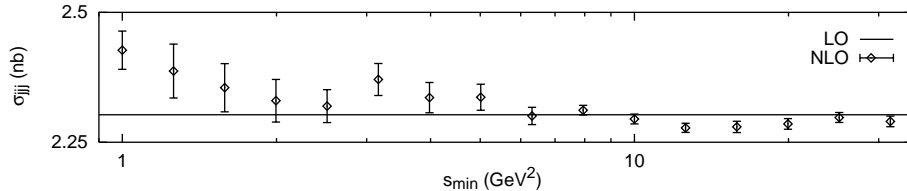


Figure 1: Next-to-leading order three jet cross section vs. s_{\min} . The leading order cross section is shown as a solid line.

line. We see that for this choice of parameters, the magnitude of the next-to-leading correction is small.

I have also computed the transverse energy spectrum of the leading jet (E_{T1}). Figure 2 shows a first attempt to explore the scale dependence of the calculation. The kinematic cuts (and parton distributions) for the results shown in figure 2 are the same as in figure 1, but the renormalization and factorization scales for the upper, middle and lower curves are chosen such that $\mu_F = \mu_R = E_{T1}/2, E_{T1}, 2E_{T1}$ respectively. For the next-to-leading order calculation, s_{\min} was chosen to be ~ 20 GeV 2 .

Comparing to the leading order results (shown as solid lines), we see that the next-to-leading corrections are indeed small, and that the scale dependence is substantially reduced.

4 Applications

The next-to-leading order calculation of three jet production will have a wide array of phenomenological applications.

4.1 Measurement of α_s

It should be possible to extract a purely hadronic measurement of α_s . One possibility for such a measurement would be a comparison of the three jet to two jet event rate. Since both processes are sensitive to all possible initial states at tree-level, a next-to-leading order comparison should be relatively free of bias from the parton distributions. Because the measurement will be simultaneously performed over a wide range of energy scales, the running of α_s can be used to constrain the fits and enhance the precision of the combined measurement.

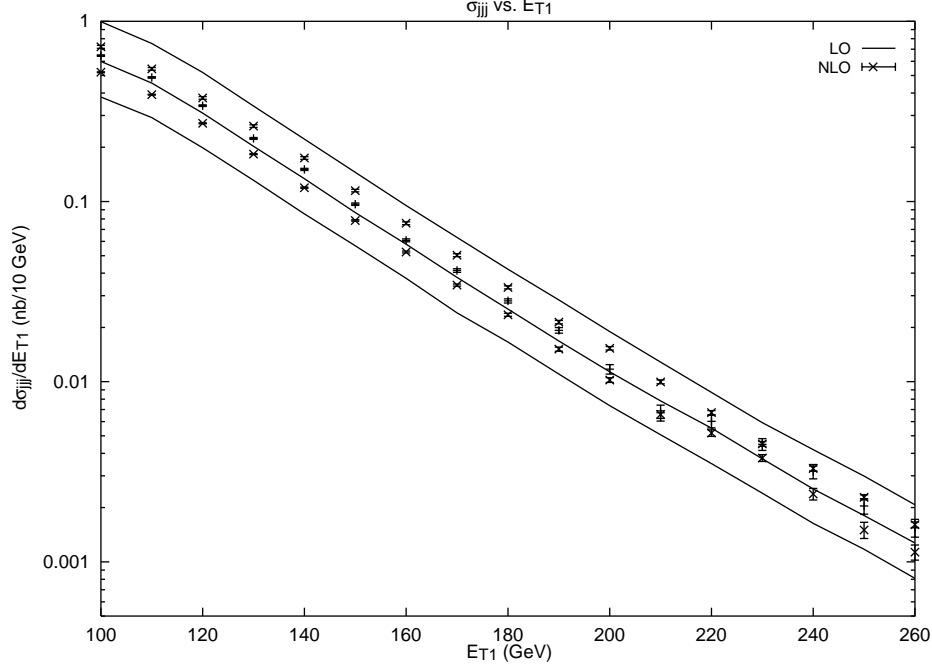


Figure 2: Transverse energy spectrum of the leading jet for different choices of scales. The next-to-leading order results are shown as points and the leading order results as solid lines. In each case, the upper points correspond to $\mu = E_{T1}/2$, the middle points to $\mu = E_{T1}$ and the lower points to $\mu = 2E_{T1}$.

4.2 Study jet clustering algorithms

Because there are up to four partons in the final state, as many as three partons can end up in a single jet. This makes the three jet calculation sensitive to the details of jet clustering algorithms. This sort of study in pure gluon production [7] uncovered an infrared sensitivity in the commonly used iterative cone algorithms.

4.3 Study jet structure and shape

Because there can be three partons clustered into a single jet, this calculation will allow truly next-to-leading order studies of the energy distribution in jets. Studies of jet production in deep inelastic scattering [8] show that the next-to-leading order correction for this variable is substantial and agrees rather well with experimental measurements.

4.4 Study event shape variables

There has been a long history of studying event shape variables like Thrust at e^+e^- colliders. These measurements challenge the ability of perturbative QCD to describe the data and provide a means (other than event rate) of obtaining a precise measurement of α_s . It will be interesting to see if one can make a meaningful study of such variables at hadron colliders.

Acknowledgments

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